Methods for modeling microwave plasma system stability

Paul Rummel[a)] and Timothy A. Grotjohn
Department of Electrical and Computer Engineering, Michigan State University, 2120 Engineering Building, East Lansing, MI 48824-1226

(Received 14 May 2001; accepted 2 January 2002)

Today's modern commercial semiconductor plasma processes require, more than ever, stable and repeatable energy delivery. One of the challenges to utilizing microwaves for plasma processing is an inherent instability that sometimes renders ''repeatable energy delivery'' difficult to achieve. This instability often manifests itself as a propensity for the plasma to extinguish or rapidly change to a lower density as system components are adjusted to facilitate optimal energy transfer from the microwave generator to the plasma. This article presents two modeling methods for demonstrating microwave powered plasma system stability, both based on simple plasma system component mathematical models. Each component of a microwave plasma system was first represented with simple equations. These individual plasma system component equations were combined into a single closed-loop 'plasma system' Matlab Simulink model. Simulation output was compared directly against actual measured operating parameters of a microwave cavity plasma reactor (MCPR). The individual plasma system component equations were also combined into a single differential ''system'' equation or ''state'' equation. This state equation was then tested with a graphical method to further illustrate characteristics of system stability. The Simulink model and the graphical control analysis clearly demonstrated major trends of actual MCPR stability performance, showing the extent to which the actual system could be perturbed before stability was lost, and that the observed instabilities appear to be primarily caused by first order effects of system component interactions. [DOI: 10.1116/1.1453454]

I. INTRODUCTION

Systems with plasma loads excited by resonant antennas, impedance matched by resonant circuits or cavities, and powered by generators of various source impedances are invariably unstable over some operating conditions. These instabilities have been documented for microwave plasma reactors in several articles as multiple steady states, hysteresis in operating states versus reactor input changes, and jumps in operating states.[1–8] For microwave systems, a common instability manifests itself as a propensity for the plasma to extinguish or rapidly change to a lower density as the impedance matching device is adjusted to minimize reflected power returning to the microwave generator.[6,7]

In practice, plasma system instabilities abound, especially in plasma processing equipment operating at low-pressure regimes or with highly coupled source designs. One manifestation of this instability as described above can make it impossible to adjust the impedance matching mechanism to obtain optimum energy transfer without extinguishing or greatly changing the plasma.[6,7,9] Though the need to control absorbed power in the plasma system to maintain process repeatability is somewhat obvious, there are also important motivations for minimizing reflected energy from the matching device/launching mechanism/plasma, other than efficiency reasons. For microwave cavity type plasma sources,[6–8] the mechanical structure of the cavity directly affects the electromagnetic fields incident upon the plasma, which influences the uniformity and density profiles of the electrons, ions, radicals, etc. At any appreciable given reflected power from the cavity, there could potentially be multiple cavity mechanical (impedance matching) positions, even keeping within the same resonance. This means that a plasma process could yield different rate/uniformity results for a given fixed amount of reflected power depending upon how the cavity is positioned or ''tuned.'' Thus, if measuring forward and reflected rf powers are the only ''diagnostic'' means for maintaining repeatable plasma densities, it is imperative that the reflected power be brought to a minimum by the matching device. For microwave powered plasma systems, however, the point of minimum reflected power is often very close to or at an unstable operating condition that leads to plasma loss or fluctuations.

In this work, direct mathematical simulation and state-model control analysis of a typical microwave plasma system are used to illustrate and predict this common instability. The developed Matlab Simulink model is a closed-loop mathematical simulation with forward power setpoint, cavity height, transmission line length as inputs, and reflected power as an output.

The developed state model that describes the plasma reactor system is a time dependent, first order, ordinary differential equation. Using a commonly used control plotting method, the degree to which the system can be perturbed before equilibrium is lost can be predicted. This is also known as the ''region of attraction.''

A microwave cavity plasma reactor (MCPR)[6–8,10] is used to first characterize the developed models, setting various proportionality constants. Data from the Simulink and state

a)Present address: COMDEL, 11 Kondelin Rd., Gloucester, MA 01930; electronic mail: paul_rummel@comdel.com

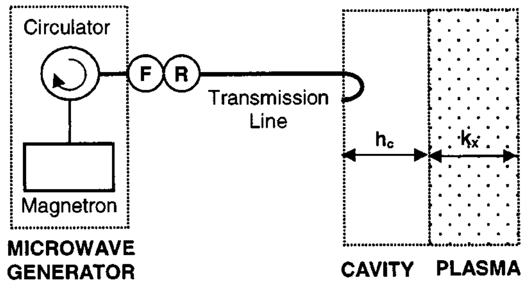

FIG. 1. Typical microwave plasma reactor system.

equation models are directly compared to MCPR stability characteristics. Then the MCPR and models are modified to alter stability performance to further test the models' correlation.

II. MICROWAVE PLASMA SYSTEM

A typical microwave plasma reactor system is shown in Fig. 1. The microwave generator is comprised of a high voltage dc power supply driving an output device such as a cavity magnetron or traveling wave tube. A circulator is used at the output of the generator to protect the output device from reflected energy, i.e., mismatch, which could cause a shortened device lifetime and unstable operation, such as a shift in output frequency. Directional couplers placed along the transmission line measure forward (F) power to and reflected (R) power from the load. Since it is often physically inconvenient to locate a bulky microwave generator directly at the plasma source, it is common practice to use a transmission line to deliver the microwave energy to the source. Since a plasma is not a fixed impedance energy load, an impedance matching device is required to facilitate maximal transfer of energy from the generator to the plasma. An adjustable cavity type launching mechanism does its impedance matching through its internal antenna structure and cavity dimensions.

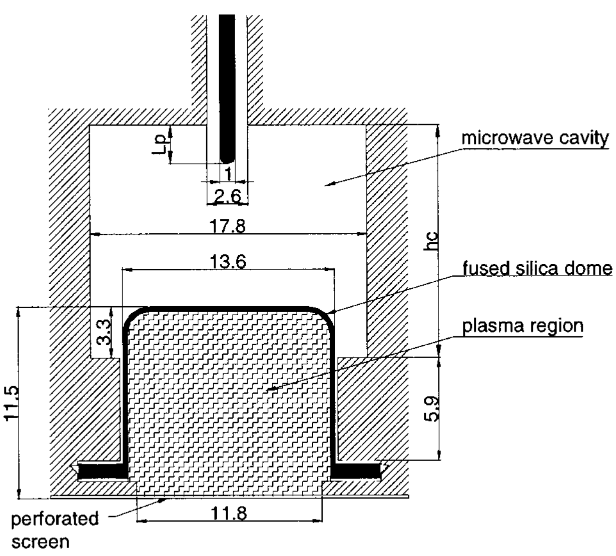

FIG. 2. MCPR with all dimensions in centimeters.

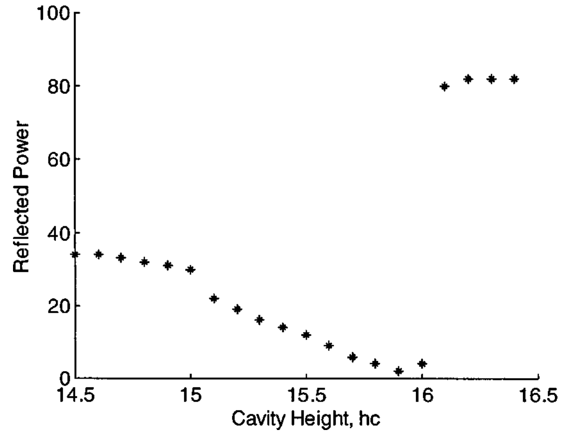

FIG. 3. MCPR system stability performance.

The experimental MCPR used in this study is shown in Fig. 2. It has a 17.8 cm diam microwave cavity with a fused silica dome on one end that confines the plasma. The diameter of the discharge is 12.5 cm. The discharge was operated with argon gas at a flow rate of 75 sccm and a pressure of 500 mTorr for this study. The probe depth L_p and the cavity height h_c can be adjusted under these conditions so that the reflected power approaches 0 W (less than 1% reflected). The 2.45 GHz microwave power generator was set for 200 W incident or forward power. A typical tuning curve is shown in Fig. 3 where the reflected power is plotted versus cavity height h_c. As the cavity height is increased starting at 14.5 cm the reflected power decreases and the power absorbed by the plasma increases. This raises the plasma density in the plasma. At a cavity height of 15.9–16.0 cm the reflected power reaches a minimum. If the cavity height is tuned past the 16 cm point, a rapid increase in the reflected power occurs and the plasma density drops. This reflected power versus cavity height curve has hysteresis. Plasma recovery (and low reflected power) is not obtained again until the cavity height is decreased well below 14.5 cm. It is this effect that the following first order models attempt to demonstrate.

III. MICROWAVE PLASMA SYSTEM MODEL

A plasma system model can be developed using the components that comprise the system, as shown in Fig. 4. The components of the Fig. 4 block diagram are developed below.

A. Plasma density model

The form of the plasma density model used is based upon energy balance equations for electropositive plasmas.[11] This form is $n_0 = k_L P_{\text{abs}}(n_0)$, where n_0 is plasma density, k_L is a plasma load line constant, and P_{abs} is absorbed power. A second term is added to represent a time dependence of plasma density to changes in absorbed power, due to collision rates, ionization rates, etc. This time constant τ can be arbitrary because it will be the only chosen time dependent term in the system equation, and this article is not concerned

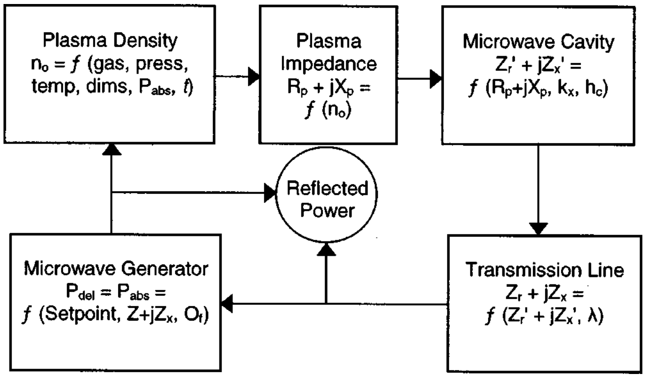

FIG. 4. Plasma system model block diagram.

with actual transient response times. The plasma density model equation now becomes the first order ordinary differential equation:

$$n_0 = k_L P_{\mathrm{abs}}(n_0) - \tau dn_0/dt.$$

Solving this equation for dn_0/dt yields:

$$d\mathbf{n}_0/dt = \mathbf{P}_{\mathrm{abs}}(n_0) k_{\mathbf{L}}/\boldsymbol{\tau} - \mathbf{n}_0/\tau.$$

Assumptions: Plasma density is linearly proportional to absorbed power P_{abs}.

B. Plasma impedance model

Plasma impedance Z_p can be defined by the series representation of $Z_p = R_p + jX_p$. As a function of plasma density n_0, the real part of the plasma impedance R_p is linear-inverse proportional to conductivity, which therefore makes it linear-inverse proportional to plasma density n_0. The expression for R_p is

$$\mathbf{R_p} = \frac{1}{(\mathbf{n}_0 + \text{cavity losses})}.$$

In this equation, "cavity losses" in cm^{-3} is a small term introduced for the purpose of eliminating the zero plasma density singularity. The reactive part of the plasma impedance will be ignored (i.e., $X_p = 0$) in this plasma impedance model since it will be absorbed into the relationship of the plasma density to the cavity reactance in the following microwave cavity model.

Assumptions: The relationship between plasma conductivity and density is linear.

C. Microwave cavity model

At the desired operating point, the microwave cavity transforms the real part of the plasma impedance to the nominal transmission line impedance Z_0, which is 50 Ω. The real part of the microwave cavity input impedance $Z_r' =$ transformation constant $(\mathbf{k}_r) * R_p$:

$$\mathbf{Z_r'} = \frac{\mathbf{k_r}}{(\mathbf{n_0} + \text{cavity losses})}.$$

In this equation, Z_r' is the real part of the impedance $Z' = Z_r' + jZ_x'$ looking into the cavity from the driving point.

As a simple model, the reactive term is modeled as a function of only two influences: (1) the effect of the plasma conductivity skin depth upon cavity resonance and (2) the effect of cavity height. These influences are treated here as having a linear effect on the cavity input reactance given by

$$\mathbf{Z_x'} = \mathbf{j}[\mathbf{k_x}(\mathbf{Z_r'} - \mathbf{Z_0})] + \mathbf{j}[\mathbf{k_h}(\mathbf{h_c} - \mathbf{h_0})].$$

The dimensionless plasma skin depth proportionality constant k_x represents the degree to which a changing plasma density affects cavity resonance. The reactive part of the cavity input impedance is approximated as changing linearly by the degree to which the real part of the cavity input impedance, Z_r' deviates from the nominal transmission line impedance Z_0. This term also absorbs any reactive part of the plasma itself as previously mentioned. The cavity height proportionality constant k_h (Ω/cm) represents the degree to which changing the cavity height affects Z_x'. Thus, the reactive part of the cavity input impedance is also approximated as changing linearly by the degree to which the cavity height h_c deviates from the optimal best tune height h_0.

Assumptions: The relationship between plasma density and cavity input reactance is inverse linear. The relationship between cavity height and cavity input reactance is linear.

D. Transmission line model

Standard lossless transmission line equations,[12] transform the impedance looking into the cavity Z' into the impedance $Z = Z_r + jZ_x$ looking into a λ wavelengths long Z_0 ohm transmission line via the following development:

$$\Gamma_v = (Z_r' + jZ_x' - Z_0)/(Z_r' + jZ_x' + Z_0)$$

(voltage reflection coefficient)

$$R_x = (\Gamma_v \cos(2\lambda) - 1)/2$$

$$R_y = \Gamma_v \sin(2\lambda)/2$$

$$\mathbf{Z_r} = \frac{\mathbf{Z_0}}{\frac{\mathbf{R_y^2}}{\mathrm{abs}(\mathbf{R_x})} - \mathbf{R_x}} - \mathbf{Z_0},$$

$$\mathbf{Z_x} = \frac{(\mathbf{Z_0 R_y})}{\frac{(\mathbf{R_x R_y^2})}{\mathrm{abs}(\mathbf{R_x})} - \mathbf{R_x^2}}.$$

Assumptions: The transmission line is lossless.

E. Microwave generator with circulator model

The system transmission line is assumed to be lossless so generator delivered power P_{del} is the same as plasma absorbed power P_{abs}. Since a circulator is typically connected at the output of the microwave generator, the delivered power P_{del} is a function of reflected power P_{rfl} given by

$$P_{\mathrm{del}} = P_{\mathrm{abs}} = P_{\mathrm{set}} - P_{\mathrm{rfl}},$$

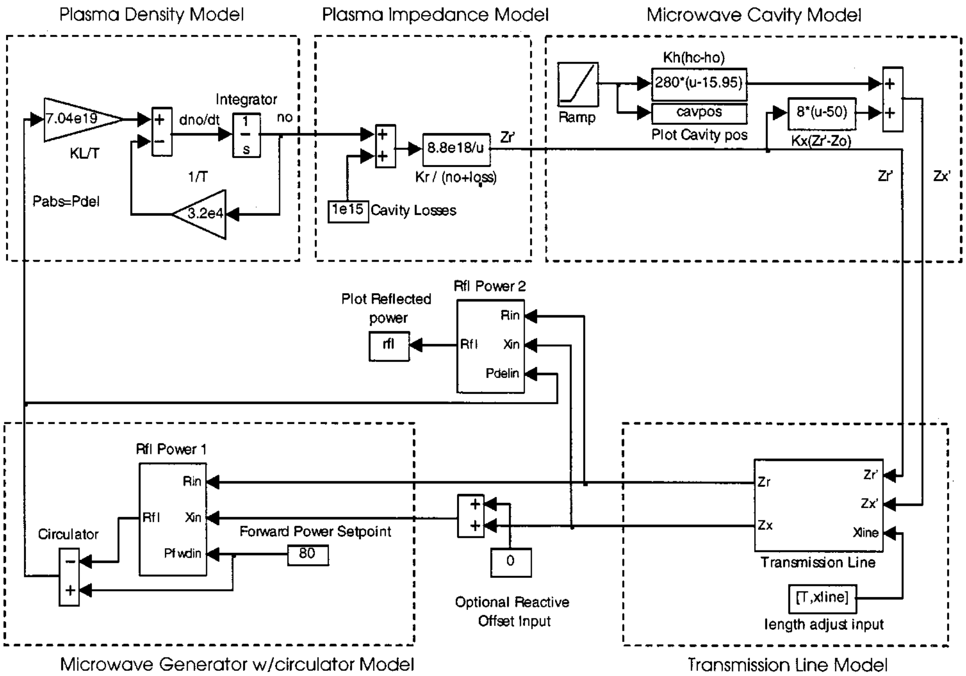

FIG. 5. Matlab Simulink model.

where P_{set} is the generator setpoint or incident power which is often automatically controlled. It can be argued that delivered or absorbed power should be a constant since P_{set} can be controlled by feedback from P_{del} or forward power P_{fwd}. Since the speed of any such power "servo" control loop would be much slower than the plasma density rate constant τ mentioned in the plasma density model above, setpoint power control cannot be used to improve system stability. Thus, any externally applied control loop effects are ignored in this analysis.

It can be shown that the complete ideal microwave generator w/lossless circulator model relating delivered output or absorbed power P_{abs} as a function of setpoint power P_{set}, nominal impedance $Z_0=50\,\Omega$, and load impedance Z_r+jZ_x (in terms of magnitudes) can be developed as

$$\text{power reflection coefficient}\quad \Gamma_p=\frac{(Z_r-Z_0)^2+Z_x^2}{(Z_r+Z_0)^2+Z_x^2}.$$

$$\text{reflected power}\quad P_{rfl}=P_{set}{*}\Gamma_p$$

$$\mathbf{P}_{abs}=\mathbf{P}_{set}-\left[\mathbf{P}_{set}{*}\frac{((\mathbf{Z_r}-\mathbf{Z_0})^2+(\mathbf{Z_{x+}}\mathbf{O_f})^2)}{(\mathbf{Z_r}+\mathbf{Z_0})^2+(\mathbf{Z_{x+}}\mathbf{O_f})^2}\right].$$

A reactive offset term O_f is added to model the effect of shifting the effective source impedance of the microwave generator to some other point away from 50 Ω real (nominal). In effect, this modifies the stability characteristics of the microwave plasma system and will be used in Sec. VI to further validate the models' ability to predict actual MCPR system stability.

Assumptions: The circulator is lossless.

F. Reflected power measurement model

Since the Simulink model will be used to directly compare its reflected power output to the actual MCPR system, the reflected power measurement will be modeled as a function of the known absorbed power P_{del} instead of setpoint power P_{set} and can be developed by

$$\text{reflected power}\quad \mathbf{P_{rfl}}=\mathbf{P_{abs}}{*}\frac{\mathbf{\Gamma_p}}{\mathbf{1}-\mathbf{\Gamma_p}},$$

where power reflection coefficient Γ_p is the same as that developed in the microwave generator with the circulator model above.

IV. SIMULINK MODEL

The preceding component equations were brought together into a single Matlab Simulink model shown in Fig. 5. The plasma system blocks of Fig. 4 are shown with dashed lines on the Simulink model. The transmission line model as well as the math models for reflected power are shown as simplified Matlab subsystem blocks for ease of readability.

To compare the Simulink model to actual data from the MCPR, the model forward power setpoint was first set to 80 W. The initial value for the integrator representing plasma density in the plasma density model was set to a starting plasma density of $1.75\times10^{17}\ \text{cm}^{-3}$. Transmission line losses and proportionality constants k_L, k_r, and k_x were determined in the following Control State Model (Sec. V). The cavity height proportionality constant k_h was adjusted (280 Ωs/cm) to match the horizontal or cavity height scale and h_0 was set to the optimal tune cavity height (15.95 cm) of the MCPR stability performance plot of Fig. 3.

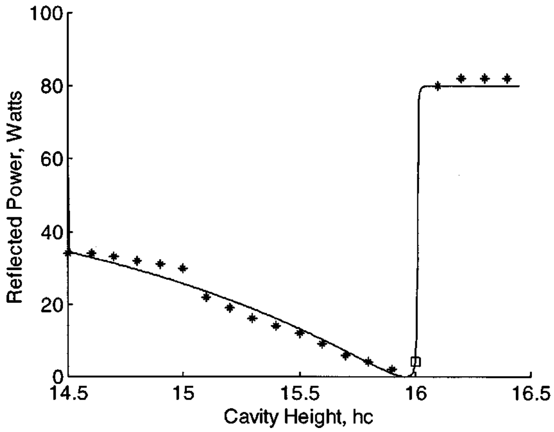

FIG. 6. Simulink model and MCPR system stability performance.

Cavity height is varied with a ramp generator in the microwave cavity model. The ramp initial output was set to 14.5 cm with a slope of 195. The simulation solver step time was set to a fixed-step size of 10 μs, with start time=0 and stop time=10 ms. The reflected power and cavity positions are plotted as the outputs. The Simulink model was named ''PlasmaSim'' and was exercised with the following simple Matlab 'M' file:

```
Title ('Plasma System Stability,' 'fontsize,' 14)
xlabel ('Cavity Height, hc,' 'fontsize,' 14)
ylabel ('Reflected Power, Watts,' 'fontsize,' 14)
T=0;          %transmission line input
xline=.01;    %transmission line input
sim PlasmaSim;  %run the simulation
plot (cavpos, rfl)  %plot the output
```

A plot similar to the MCPR stability performance plot of Fig. 3 is thus created. The output of the Simulink model is overlaid on Fig. 3 and is shown in Fig. 6. The solid line represents the Simulink model data and clearly mimics the major trend of MCPR stability. As the model's cavity height input is raised beyond 16 cm the plasma is extinguished (plasma density goes to zero) and the simulated reflected power rapidly goes to the 80 W setpoint. For both the Simulink model and actual data of the MCPR, the crossover point from stability to instability happened at approximately 4 W as indicated by the square data point of Fig. 6.

Differences in data between the Simulink model and actual MCPR data could be due to a number of factors beyond instrumentation errors:

(1) The simplistic linear-inverse conductivity proportionality of the plasma impedance model. In the real MCPR system, the shape of the plasma and its relationship to the electromagnetic fields of the cavity surely change as a function of plasma density.

(2) The model's assumption that the transmission line is lossless. In operating the MCPR system, the setpoint power was actually 200 W. Using coaxial cable loss data, it was calculated that if all energy reaching the cavity was reflected, only 80 W would reach the reflected power meter located back at the microwave generator. This is also confirmed by actual maximum reflected power being 80 W. This is the basis for using 80 W for the Simulink model power setpoint. In reality, coaxial cable losses would not be constant as a function of cavity input impedance because, at 2.45 GHz and 50 Ω, cable dielectric losses are not equal to cable copper losses.

(3) Nothing in the simple microwave cavity model accounts for how input impedance is affected by cavity mode changes or surface irregularities.

(4) The circulator was modeled as ideal and lossless which is not true in reality.

Thus, to closer model the real system, the complexity of the component math models must increase to include these secondary effects.

V. CONTROL STATE MODEL

A commonly used method for determining the stability of control systems is to develop and analyze differential equations that describe the system. The individual component models described above can be combined into a single, first order, ordinary differential equation. Analyzing the stability of the plasma system can then be accomplished by observing the zeros and plotted curve of this system differential equation. The zeros of the curve are termed ''equilibrium states.'' For any first order system such as this, *solving* the differential equation is not required to predict its response. It is enough to simply plot the equation to determine the equilibrium states, determine whether the states are stable or not, and then determine the ''region of attraction'' for the stable states. This region indicates how much the system can be perturbed before it loses equilibrium[13] (becomes uncontrollable). The form of the single order differential equation for state-model analysis is $dn_0/dt=f(n_0)$.

By combining the individual plasma system component models outlined above and solving for the rate of plasma density change dn_0/dt, the differential equation shown below in Eq. (1) is obtained. First, the equation for P_{abs} of the microwave generator model is substituted for P_{abs} in the plasma density equation. Then, Z_r and Z_x of the microwave cavity model are substituted for Z_r and Z_x of the microwave generator model. The resulting equation becomes a function of constants and n_0.

$$x'=\frac{[2Z_0k_xO_f-Z_0^2(1+k_x^2)-O_f^2]x^3-[2Z_0k_r(1-k_x^2)+2k_xk_rO_f]x^2+[4Z_0k_Lk_rP_{\text{set}}-k_r^2(1+k_x^2)]x}{[Z_0^2(1+k_x^2)-2Z_0k_xO_f+O_f^2]x^2+[2Z_0k_r(1-k_x^2)+2k_xk_rO_f]x+[k_r^2(1+k_x^2)]}. \tag{1}$$

The variable x is used for the state variable n_0, and x' for dn_0/dt. The cavity height h_c was assumed to be fixed in the optimal position $h_c=h_0$, hence k_h does not appear in the equation. The transmission line component equations were not used because they would make the state equation overly complex and, in this case, do not affect system stability. The constants used in the equation were obtained from data for the microwave cavity plasma reactor shown earlier in Fig. 2 operating with the conditions described in Sec. II. The parameters k_L, k_r, and k_x were determined from the experimental data shown in Fig. 3 and plasma density measurement data taken for the MCPR plasma reactor:[6–8,10]

(i) The plasma load line constant $k_L=2.2\times10^{15}$ $\mathrm{cm}^{-3}\,\mathrm{W}^{-1}$ was determined from MCPR measured plasma density at a given absorbed power by $k_L=n_0/P_{\mathrm{abs}}$. P_{abs} was set for 80 W. A Langmuir probe was used to measure plasma density at optimal tune $n_0=1.75\times10^{17}$ cm^{-3}.

(ii) Cavity transformation constant $k_r=8.8\times10^{18}$ $\Omega\,\mathrm{cm}^{-3}$ was determined from MCPR measured plasma density for $Z_0=50\,\Omega$ by: $k_r=n_0^*Z_0$.

(iii) Skin depth proportionality constant $k_x=8$ was determined from MCPR plasma density data at the point where stability was lost. Using differential Eq. (1) above, plasma density motion x' was set to zero indicating the unstable equilibrium point. Plasma density x was set to the plasma density at the point just before the instability, 1.65×10^{17} cm^{-3}, and Eq. (1) was solved for k_x.

(iv) $P_{\mathrm{set}}=80$ W is determined from MCPR measured forward power minus estimated transmission line losses. Since the Simulink model uses lossless transmission line equations, it was desired that the model's reflected power output match the actual scale of the MCPR reflected power readings. P_{set} for the model was calculated by: (−loss factor*2* line length)=10 log [$P_{\mathrm{model\ set}}/P_{\mathrm{actual\ set}}$]. Solving for $>P_{\mathrm{model\ set}}=P_{\mathrm{actual\ set}}$* 10ˆ [−(loss factor *2* line length)/10]. Loss factor for coaxial cable used @2.45 GHz=9 dB per 100 ft=0.09 dB per foot. Coaxial cable line length=22 ft. (2* line length=total distance from generator to cavity and back to reflected power meter). $P_{\mathrm{model\ set}}$=200 W* 10ˆ [−(0.09 dB ft^{-1} *2* 22 ft)/10]=80 W.

(v) $Z_0=50\,\Omega$ is the standard transmission line nominal impedance.

(vi) $O_f=0\,\Omega$ indicates no reactive offset is included in the initial analysis. This term will be used in Sec. VI to alter the stability characteristics of the actual and modeled plasma systems to provide more data to verify the models.

A direct plot of x' versus x of state model equation Eq. (1) is given in Fig. 7, indicating the equilibrium states and region of attraction for the modeled microwave cavity plasma system. Equilibrium states are shown where the curve meets the zero plasma density velocity axis and are thus marked with round or square dots.

The stability of each equilibrium state (or point) is tested by observing the slope of the trajectory at the point. A negative slope indicates an asymptotically stable point and a positive slope is an unstable point. Thus the equilibrium points at $n_0=0$ and 1.75×10^{17} cm^{-3} are stable (round), and the point

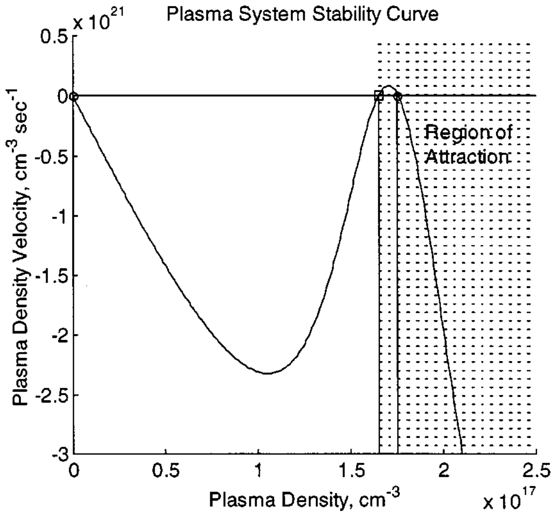

FIG. 7. Stability plot obtained from Eq. (1).

at 1.65×10^{17} cm^{-3} is unstable (square). The system state will move to the right if the state is positive in the x' direction (vertical axis), and will move to the left if the state is negative in the x' direction. Thus, as long as the operating conditions yield a plasma density above 1.65×10^{17} cm^{-3}, the system will move to the stable operating point 1.75 $\times10^{17}$ cm^{-3}. This is the region of attraction for the system and is shown as the shaded area. From the stable operating point of $n_0=1.75\times10^{17}$ cm^{-3}, a 6% drop or more in plasma density results in the system transiting to the other stable equilibrium point $n_0=0$ at a rate governed by plasma diffusion mechanisms, and the plasma is extinguished. The unstable equilibrium point represented by a plasma density 6% below the desired operating point also represents a 6% drop in absorbed power, as plasma density is modeled as a linear function of absorbed power P_{abs}. Since the incident or forward power is fixed, this also represents a 6% (of incident) rise in reflected power. This "edge of stability" point is observed at the square data point of Fig. 6. At about 4 W of reflected power (5% of 80 incident watts), the system went unstable and the plasma extinguished.

VI. SYSTEM MODIFICATION

The stability characteristics of the MCPR system were modified to see how well the models would further correlate to MCPR system behavior. The effective source impedance of the microwave generator was modified through the reactive offset O_f introduced in the microwave generator model. The actual MCPR system was modified by inserting a reactive offset at the output of the generator, between the circulator and the power meters as illustrated in Fig. 8. A coaxial shorted stub off of a coaxial 'T' was used to create a reactive offset of $O_f=-j110\,\Omega$. The shorted stub was adjusted to this reactive offset value at 2.45 GHz using a network analyzer. For operation where reflected power measured at R in Fig. 8 is at a minimum, the microwave generator now sees a reactive impedance and some power is lost to the circulator's dummy load. To maintain the same power delivered to the

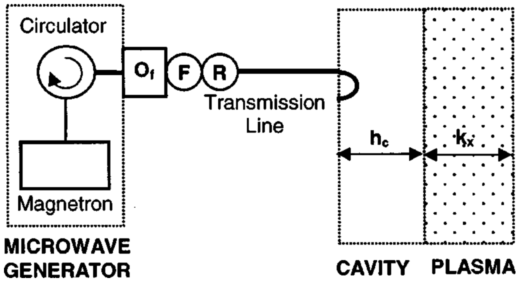

FIG. 8. Modified plasma system with reactive offset impedance added at the circulator output.

cavity and hence the same plasma density, the power setpoint of the generator had to be increased. For the models, the incident setpoint power was increased from 80 to 188 W.

Figure 9 shows the tuning curve for this modified MCPR system with the reactive offset term added to the Simulink model. This curve was generated with all the same operating conditions of gas flow, pressure, and delivered power used earlier for the original system performance of Fig. 3. In terms of region of attraction, this is a large improvement over the original system, which allowed very little deviation from the minimum reflected power operating point. Figure 10 shows the stability plot of the system state equation with the offset term O_f set to -110. By direct observation, the plasma density can be perturbed all the way down to approximately 0.9×10^{17} cm^{-3} before the system goes unstable. This represents an approximate 50% drop in plasma density and hence, absorbed power.

It should be noted that the length of the transmission line becomes a critical factor in this modified system. The modified system and Simulink model used an effective transmission line length (which was close to zero) that yielded optimal stability.

A separate family of stability curves was generated to see the effect that plasma skin depth has upon system stability as shown in Fig. 11. Using the state model differential equation with no reactive offset, the curves in Fig. 11 show how the

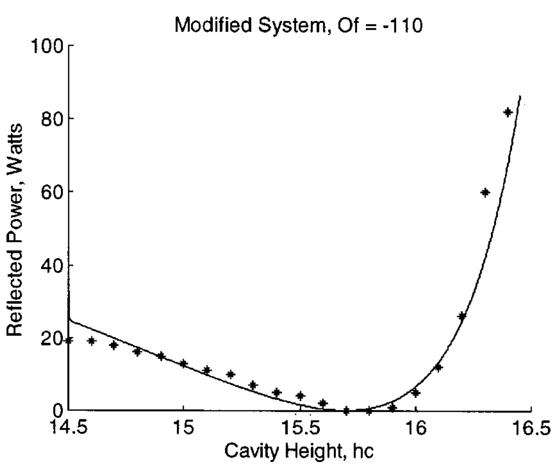

FIG. 9. Modified system performance with cavity height perturbation. (*) indicates experimental data and the solid line is the Simulink model result.

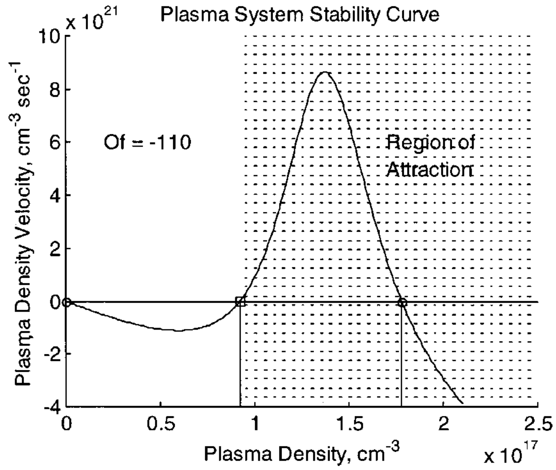

FIG. 10. Stability plot for $O_f = -j110\ \Omega$.

plasma conductivity skin depth coefficient k_x affects system stability. It is observed how a much larger region of attraction would result if changes in skin depth affected cavity resonance to a smaller degree. As shown above, reducing k_x to a value of three would allow the plasma density n_0 to be perturbed all the way down to 1.0×10^{17} cm^{-3} before the system goes unstable. It is also observed that the system would be completely stable for all states if the conductivity skin depth affects cavity resonance very little $(k_x \leqslant 1)$.

VII. CONCLUSIONS

Since both the Simulink model and the state equation were based upon the same component math models, they were expected to demonstrate similar stability characteristics. The Simulink model required specialized software, relatively long development time, and provided high flexibility for experimentation and parameter observance. The control state model provided the same pertinent stability information with very little computational energy required.

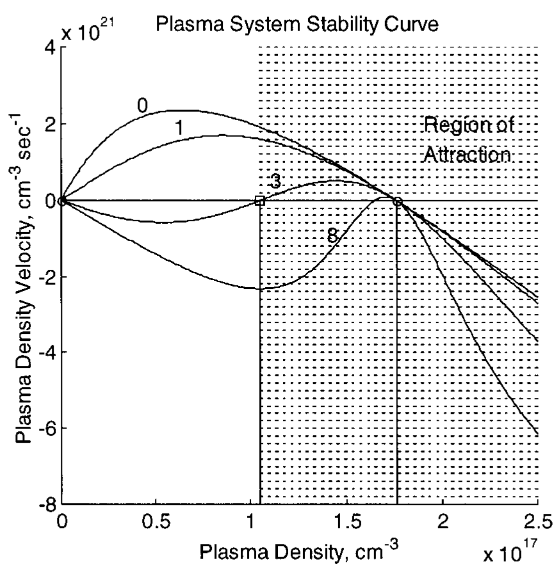

FIG. 11. Stability performance for $0 < k_x < 8$.

Modeling plasma system stability to demonstrate interactions of plasma system components is clearly shown in the preceding analysis. With all of the seemingly gross model approximations, stability correlation was still reasonably well achieved. The models showed relatively close agreement to actual data measured on an MCPR plasma system. As the actual MCPR system was modified to change its stability characteristics, the models' close correlations held. Using these modeling methods to predict ways to improve stability, it was observed that the degree to which plasma density (via microwave skin depth changes) affects cavity resonance is an important factor. Plasma source designs with resonances less affected by plasma conductivity skin depth would theoretically be more stable.

It is concluded that stability modeling of a plasma system from either direct simulation or a control state equation point of view are valid approaches. The ''component'' modeling approach leaves flexibility for rapidly predicting the results of changing real components or designs. The goal of the study was to see if first order effects would be sufficient to demonstrate major trends of plasma system stability. Clearly, the state model approach would rapidly become cumbersome as more accurate component models were used. The Simulink model might be the preferred method of using more complex components to model more complex plasma system behaviors.

[1] D. A. Carl, M. C. Williamson, M. A. Lieberman, and A. J. Lichtenberg, J. Vac. Sci. Technol. B **9**, 339 (1991).
[2] P. K. Shufflebotham and D. J. Thomson, J. Vac. Sci. Technol. A **8**, 3713 (1990).
[3] O. A. Popov, J. Vac. Sci. Technol. A **8**, 2909 (1990).
[4] S. M. Gorbatkin, L. A. Berry, and J. B. Roberto, J. Vac. Sci. Technol. A **8**, 2893 (1990).
[5] E. S. Aydil, J. A. Gregus, and R. A. Gottscho, J. Vac. Sci. Technol. A **11**, 2883 (1993).
[6] J. Asmussen and P. Mak, Rev. Sci. Instrum. **65**, 1753 (1994).
[7] P. Mak and J. Asmussen, J. Vac. Sci. Technol. A **15**, 154 (1997).
[8] P. Mak, M.-H. Tsai, J. Natarajan, B. L. Wright, T. A. Grotjohn, F. M. A. Salam, M. Siegel, and J. Asmussen, J. Vac. Sci. Technol. A **14**, 1894 (1996).
[9] A. J. Hatch and L. E. Heuckroth, J. Appl. Phys. **41**, 1701 (1970).
[10] J. Asmussen, T. A. Grotjohn, P. Mak, and M. A. Perrin, IEEE Trans. Plasma Sci. **25**, 1196 (1997).
[11] M. A. Lieberman and A. J. Lichtenberg, *Principals of Plasma Discharges and Materials Processing* (Wiley, New York, 1994).
[12] R. A. Chipman, *Theory and Problems of Transmission Lines* (McGraw-Hill, New York, 1968).
[13] H. K. Kahlil, *Nonlinear Systems*, 2nd ed. (Prentice–Hall, Englewood Cliffs, NJ, 1996).